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EVALUATING TIME SERIES PREDICTION MODELS: EGG PRICES IN MEXICO

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ABSTRACT

Mexico is the world's largest consumer of eggs, producing 3.05 million Mg in 2021. The high variation in wholesale prices is a feature of the egg production system, which is important to producers and government institutions that need to forecast future prices for activity planning. As a result, it is necessary to propose tools that can reliably predict egg prices. The goal of this paper was to compare the performance of various statistical models by analyzing the time series of egg prices using the Akaike index and forecast error to determine which model best predicts the wholesale price of white eggs. The models evaluated were the autoregressive integrated moving average model (ARIMA), ARIMA with interventions, ARIMA with transfers, and regression with ARIMA errors. Two time series were used: the wholesale price of white eggs, constructed with data from the National System of Information and Market Integration (SNIIM) and the Agrifood and Fisheries Information Service (SIAP), and egg imports, calculated with data from the Economic Information System. The latter was used as an exogenous variable to explain the price of eggs. Both cover the period from January 2006 to December 2021. According to the Akaike index, the model with the best adjustment was ARIMA (0,1,1)(1,0,1)[12] with interventions. In the evaluation of forecast error, the best models were the regression models with ARIMA (1,1,0)(1,0,1)[12] and ARIMA (1,1,0)(1,0,1)[12] errors with transfer.

Keywords: ARIMA, regression with ARIMA errors, ARIMA with interventions, ARIMA with transfers.

INTRODUCTION

In Mexico, eggs are a basic product in the food basket and one of the cheapest protein sources available. The country is the world's largest egg consumer, with a per capita consumption of 24 kg. During 2021, Mexico ranked sixth as the world's largest egg producer, with a production volume of 3.05 million Mg. The state of Jalisco was the main producer, with 1.6 million Mg. The country is the seventh-largest importer of eggs for human consumption (SIAP, 2022).

One of the characteristics of the egg product system in Mexico is the high variability in wholesale egg prices across the country. For example, in September 2006, when the weighted average price in the country was \$9.72 MXN kg^{-1}, the state of Colima recorded prices of \$7.00 MXN kg^{-1}, while Querétaro reached \$13 MXN per kilogram, which represents a variance of \$2.39 MXN. In another case, in March 2020, the average wholesale price of white eggs in the country was \$34.91 MXN kg^{-1}; in the state of Guerrero, prices of \$28 MXN kg^{-1} were recorded, while in Veracruz they reached \$64.57 MXN kg^{-1}, with a variance of \$42.12 MXN for the month (SNIIM, 2022).

The high variation makes it difficult to predict egg prices, which has an impact on producers and government institutions that need to estimate forward prices in order to plan their activities. It would therefore be useful to have a methodology for price prediction with some degree of reliability. Statistical models are among the most effective and, therefore, the most widely used alternatives for time series forecasting. Among the most widely used are autoregressive (AR), moving average (MA), autoregressive with moving averages (ARMA), autoregressive integrated with moving averages (ARIMA), and the Holt Winter smoothing model (Brockwell and Davis, 2002).

ARIMA models have been used to forecasts in a variety of research areas. For example, Alburquerque and de Moraes (2007) forecast the average monthly cocoa price paid to producers in Brazil. The analysis of the variable with ARIMA models helps investors and producers form expectations about the future price behavior of this commodity. Sánchez-López *et al*. (2013) found a decrease in future bovine milk production in Baja California, Mexico, and recommended that government agencies generate forecasts that allow producers to take technical measures to deal with these circumstances, thereby improving their response capacity.

The diversity of existing models for time series forecasting generates a constant search to improve, evaluate, and propose new models with the aim of identifying the one with the best predictions for a given case. Box and Tiao (1975) improved the traditional ARIMA model by incorporating the effects of natural or induced interventions into the study variable, such as economic shocks, weather events, pandemics, and administrative changes, or by transferring the performance of another variable over time that may influence the direction of the time series. This is known as ARIMA models with interventions, ARIMA models with transfers, and regression with ARIMA errors, and their use can help improve the forecasting accuracy of the traditional ARIMA model.

Nowadays, productive, commercial, and service activities generate and store a large amount of information to analyze and use in decision-making. It can be generated at regular time intervals, either daily, weekly, monthly, or annually. Such information is commonly referred to as a time series, in which patterns of behavior can be identified and their short- to medium-term values predicted. Models for time series are used to obtain future values for a dataset, predictions supported by historical information on the series. In this sense, the models developed by Box and Jenkins (1973) have been used to make short-, medium-, and long-term predictions in stationary series, i.e., the mean, variance, and covariance are constant over time (Tsay, 2010). The stages of the Box-Jenkins methodology correspond to stationarity verification, differencing (if required), parameter identification and estimation, residual diagnosis, and prediction (Hanke and Wichern, 2010).

This method has been used extensively. For example, in Mexico, a study was conducted for ball tomato price forecasting with a time series from December 2008 to November 2009. The study used the Box-Jenkins methodology to identify an econometric ARIMA model, which has two autoregressive factors and one moving average (Marroquín-Martínez and Chalita-Tovar, 2011). In Venezuela, a statistical model was used to analyze inflation with a time series covering the period from June 1995 to July 2000; applying the Box-Jenkins methodology, the model that best fit the time series was the ARIMA with a confidence interval of 95 % (Seijas, 2002). Luis-Rojas *et al.* (2019, 2022) used ARIMA models for the prediction of the egg price paid to the producer, taking sorghum and maize production as auxiliary variables.

For the case of climatic events, a time series study was conducted in Peru with the monthly rainfall of two different regions using the ARIMA model; depending on the region, the ARIMA structure of the rainfall of the fluviometric stations can be different or similar (Reyes, 2014). In another study comparing energy demand in India, the Metabolic Grey model (MGM), ARIMA, their combination (MGM-ARIMA), and the back propagation neural network (BP) model were used, with the intention of applying them to policies to improve energy security. It was concluded that all four models have an accuracy of more than 95 % confidence. The most reliable model was the BP, and the MGM-ARIMA model was more accurate than the separate MGM and ARIMA models (Jiang *et al*., 2018). With these methodologies, time series information can be analyzed and explained, patterns can be identified, and variables of interest in natural, social, financial, or economic phenomena can be predicted.

MATERIALS AND METHODS

A time series of weighted wholesale white egg prices was used (Yt). As an exogenous variable, the time series of egg imports into Mexico was used (X_t). Statistical analysis was performed in R software version 4.2.0.

Weighted wholesale white egg price database

The time series contains 192 monthly national wholesale price data points per kilogram of white egg from January 2006 to December 2021. This base is constructed from the average month-end wholesale prices of white eggs in the country's main supply centers recorded in the National System of Market Information and Integration (SNIIM) for the aforementioned period (SNIIM, 2022). The weighting of the wholesale price of white eggs was based on the monthly production volume per state, i.e., the prices of supply centers in states with higher production are weighted more heavily than those in states with lower production. Production data by state are from the Agricultural, Food, and Fisheries Information Service (SIAP).

Mexican import database

The import database was constructed with information from the Economic Information System (Banco de México, 2022). This includes the sum of imports of birds' eggs with and without shells (egg yolks, fresh, dried, cooked by steaming or boiling in water, molded, frozen, or otherwise preserved). This base contains the same number of observations as the time series of weighted egg prices over the same period. The data is measured in thousands of dollars. Because of the correlation between the two variables, it was decided to include shell-on and shell-off poultry egg imports in the time-series analysis of the wholesale white egg price. The higher the number of egg imports, the lower the expected egg price.

Methodology and models

The methodology proposed by Box and Jenkins (1973) applied to ARIMA models—regression with ARIMA errors, ARIMA with interventions, and ARIMA with transfer—was used in order to find the model that best fit the white egg price series according to its Akaike index and forecast error.

ARIMA (p,d,q)(P,D,Q)s models

The ARIMA (p,d,q)(P,D,Q)s models, in their most general form, arise by merging the non-seasonal and seasonal ARIMA models:

$$\phi_p(B)\Phi_P(B^s)\nabla^d\nabla_s^D Z_t = \delta + \theta_q(B)\Theta_Q(B^s)a_t,$$

where Z_t is the response variable, δ is a constant, $\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p)$ is the non-seasonal autoregressive term, $\Phi_P(B^s) = (1 - \Phi_1 B^s - \Phi_2 B^{2s} - \cdots - \phi_p B^{Ps})$ is the seasonal autoregressive term, $\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q)$ is the non-seasonal moving averages term, $\Theta_Q(B) = (1 - \Theta_1 B^s - \Theta_2 B^{2s} - \cdots - \theta_Q B^{Qs})$ is the seasonal moving averages term, $\nabla^d = (1 - B)^d$ is the differencing of the non-seasonal part with d differencing, $\nabla_s^D = (1 - B^s)^D$ is the differencing of the seasonal part with D differencing, and α_t is the white noise (Tsay, 2010).

Regression with time series errors

In many applications, the relationship between two time series is particularly interesting if it leads to the consideration of a linear regression. In the standard linear regression, the errors are independent and identically distributed. However, in various applications of regression analysis, this assumption does not hold true. It is often more appropriate to assume that the errors come from a stationary second-order process with zero-mean. Since the errors can be approximated by an appropriately chosen ARMA (p,q) process, it is of particular interest to consider the following model:

$$Y_t = \alpha + \beta X_t + e_t,$$
$$e_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + a_t - \sum_{i=1}^{q} \theta_i a_{t-i}$$

where Y_t represents the dependent variable at the time t that is intended to be predicted, α is the coefficient of intersection in linear regression, β is the coefficient of the independent variable X_t in linear regression, and e_t reflects the discrepancy between the actual observation and the model prediction.

Within the ARMA process, ϕ_0 is the constant coefficient on the autoregressive (AR) component; ϕ_i is the coefficient of the autoregressive terms, with i varying from 1 to p; the terms Y_t are the past values of the variable over time Y; t-i is the error term over time α_t ; t is the coefficient of the moving average terms, with θ_i varying from 1 to q; and the term α_{t-i} is the past values of the error terms over time t-i (Brockwell and Davis, 2002).

ARIMA models with interventions

This methodology was developed by Box and Tiao (1975) and is used to model interventions or changes in time series. The effects of exogenous variables ξ_t (impulses or level changes) can be modeled as follows:

$$Y_t = \sum_{i=1}^{n} \frac{\omega_i(B)}{\delta_i(B)} \xi_{ti} + N_t,$$

where δ_i and ω_i represent n previously lumped parameters of the dynamic transfer of ξ_{t-i}, B is the lag operator, and N_t is the error, which is modelled as a time series model as follows:

$$N_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + a_t - \sum_{i=1}^{q} \theta_i a_{t-i}$$

In these models, ξ_t is considered as a deterministic or intervening variable, which is generally associated with two types of effects: impulse, taking a value of one when the impulse happens in time ξ_t and zero when the impulse does not happen:

$$\xi_t \equiv \begin{cases} 1 \ if \ t = T \\ 0 \ if \ t \neq T \end{cases}$$

and step, which takes a value of one when the level or step change occurs in time T and zero when the level change does not occur:

$$\xi_t \equiv \begin{cases} 1 \; if \; t \geq T \\ 0 \; if \; t < T \end{cases}$$

ARIMA models with transfer

An important type of dynamic relationship between an output Y_t that is explained by an input X_t (each measured in equispaced times) is one in which the deviations of the input and output from the appropriate mean values are related by a linear difference equation (Reinsel, 1997):

$$\begin{aligned} (1-\delta_1 B - \cdots - \delta_r B^r)Y_t &= (\omega_0 - \omega_1 B - \cdots - \omega_s B^s)X_{t-b} \\ (1-\delta_1 B - \cdots - \delta_r B^r)Y_t &= \left(\omega_0 B^b - \omega_1 B^{b+1} - \cdots - \omega_s B^{b+s}\right)X_t \\ \delta(B)Y_t &= \omega(B)B^b X_t \\ \delta(B)Y_t &= \omega(B)X_{t-b} \end{aligned}$$

Alternatively, the output Y_t and input X_t can be said to be linked by a linear filter:

$$\begin{aligned} Y_t &= \upsilon_o X_t + \upsilon_1 X_{t-1} + \upsilon_2 X_{t-2} + \cdots \\ &= \upsilon(B)X_t, \end{aligned}$$

where the transfer function $\upsilon(B) = \upsilon_0 + \upsilon_1 B + \upsilon_2 B^2 + \ldots$ can be expressed as a ratio of two polynomial operators:

$$\upsilon(B) = \delta^{-1}(B)\, \omega(B)\, X_{t-b}$$

However, the problem of estimating with the model, linking an output Y_t and an input X_t, is complicated in practice by the presence of noise N_t, which corrupts the true relationship between input and output according to the following equation:

$$Y_t = \upsilon(B)\, X_t + N_t$$

where X_t and N_t are independent processes and N_t can be described as a stationary stochastic process as an ARMA (p,q) model:

$$N_t = \phi_0 + \sum_{i=1}^{p} \phi_i Y_{t-i} + a_t - \sum_{i=1}^{q} \theta_i a_{t-i}$$

In practice, it is necessary to estimate the transfer function of the linear filter describing the noise, in addition to the transfer function $\upsilon(B) = \delta^{-1}(B)\, \omega(B)\, X_{t-b}$ describing the dynamic relationship between the input and output (Box *et al*., 2016).

Akaike index

The Akaike information criterion (AIC) is defined as:

$$AIC = \frac{-2}{T}\,\text{In (probability)} + \frac{2}{T}\text{(number of parameters)}$$

where the likelihood function is evaluated in maximum likelihood estimates and T is the sample size (Akaike, 1974).

Cross-validation

Cross-validation generally uses three statistics to measure the performance of point forecasts: the root means square error (RMSE), the mean absolute deviation (MASE), and the mean absolute percentage error (MAPE). For step-by-step forecasts, these measures are defined as follows:

$$RMSE(\ell) = \sqrt{\frac{1}{m}\sum_{j=0}^{m-1}\left[x_{T+\ell+j} - x_{T+j}(\ell)\right]^2},$$

$$MASE(\ell) = \frac{\frac{1}{m}\sum_{j=0}^{m-1}\left|x_{T+\ell+j} - x_{T+j}(\ell)\right|}{Q},$$

$$MAPE(\ell) = \frac{1}{m}\sum_{j=0}^{m-1}\left|\frac{x_{T+j}(\ell)}{x_{T+j+\ell}} - 1\right|,$$

where m is the number of available step-by-step forecasts in the forecast sub-sample and Q is a scaling constant. Regularly, the model with the lowest value on these measures is considered the best forward forecasting model (Diebold and Mariano, 2002; Hyndman and Koehler, 2006; Kim and Kim, 2016).

RESULTS AND DISCUSSION

Series characteristics

The time series of monthly wholesale white egg prices in Mexico covers a period from January 2006 to December 2021 (192 observations). The latter presents a pattern of growth and variability with three factors: trend, seasonality, and a random component, also known as white noise. The seasonality of the wholesale white egg price series throughout the year shows that the months of February, March, and December have the highest average values for this product. The lowest price was reached in June.

Transformations and augmented Dickey-Fuller test

To achieve variance stationarity, a logarithmic transformation was applied to the egg price series. The augmented Dickey-Fuller test (Cheung and Lai, 1995) examines the hypotheses H_0 (series has a unit root) and H_a (series does not have a unit root) at a significance level of α = 0.05 to determine whether a series is stationary or not. By using the adf.test() function in the R statistical package on the transformed series, a value of $p = 0.1706$ was obtained, which is greater than α. Therefore, H_0 is not rejected, and the series is concluded to have a unit root. As a result, the series needed to be differentiated and then retested.

Taking the first difference, a stationary series in variance and level is obtained, which is confirmed by the augmented Dickey-Fuller test. When applying the test in R, $p = 8.2879 \times 10^{-12}$, which is less than the significance level of α = 0.05. Therefore, H_0 is rejected, and the series is found to be stationary.

Autocorrelation (ACF) and partial autocorrelation (PACF) graphs

ACF and PACF charts are useful tools for analyzing time series properties and determining the appropriate ordering of an ARIMA model (Box and Pierce, 1970; Velicer, 1976). The graphs of the series with the logarithmic transformation of the wholesale white egg price in Mexico, obtained using the ggAcf() function of the forecast package in R (Figure 1), show a significant correlation at lag 12. This suggests that the series has a seasonal component, as the correlation is high at 12-month intervals.

Selection of the candidate ARIMA models

Based on the ACF and PACF plots of the time series, ARIMA models (Table 1) are proposed, which include regression models with ARIMA errors, ARIMA with interventions, and ARIMA with transfers.

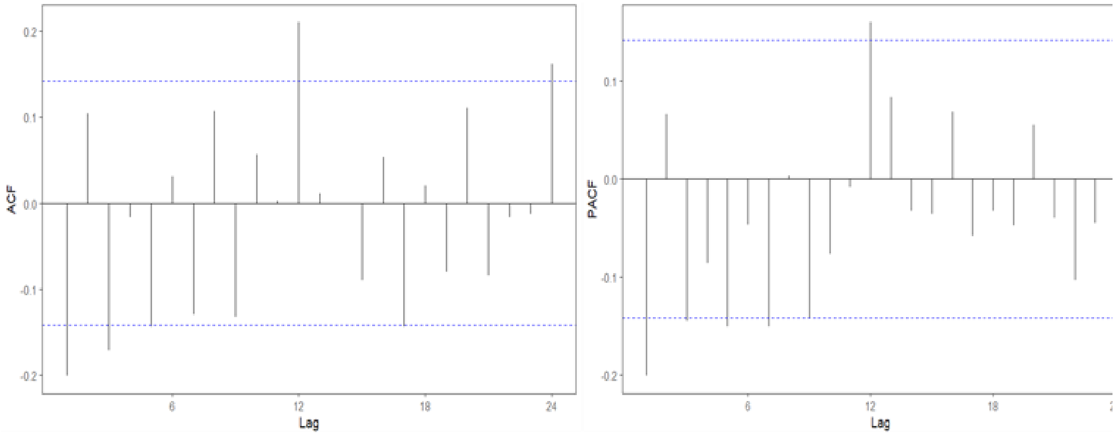

Figure 1. Autocorrelation (ACF) and partial autocorrelation (PACF) graphs for the determination of the model.

Table 1. Proposed ARIMA models to analyze the time series.

Models
ARIMA (4,1,3)(1,0,1)[12]
ARIMA (3,1,3)(1,0,1)[12]
ARIMA (3,1,2)(1,0,1)[12]
ARIMA (3,1,1)(1,0,1)[12]
ARIMA (3,1,0)(1,0,1)[12]
ARIMA (2,1,3)(1,0,1)[12]
ARIMA (2,1,2)(1,0,1)[12]
ARIMA (2,1,1)(1,0,1)[12]
ARIMA (2,1,0)(1,0,1)[12]
ARIMA (1,1,3)(1,0,1)[12]
ARIMA (1,1,2)(1,0,1)[12]
ARIMA (1,1,1)(1,0,1)[12]
ARIMA (1,1,0)(1,0,1)[12]
ARIMA (0,1,3)(1,0,1)[12]
ARIMA (0,1,2)(1,0,1)[12]
ARIMA (0,1,1)(1,0,1)[12]

ARIMA model adjustment

Estimating the coefficients of the candidate ARIMA models

To estimate the parameters of the proposed ARIMA models, the arima() function from the forecast package in R was used. Models with parameters that did not meet the 0.05 significance level were discarded. There were only four models with all significant coefficients: ARIMA (3,1,1)(1,0,1)[12], ARIMA (1,1,3)(1,0,1)[12], ARIMA (1,1,0)(1,0,1)[12] and ARIMA (0,1,1)(1,0,1)[12].

Diagnosis of candidate ARIMA models

To check for white noise in the residuals, the ACF plot and the Ljung-Box test, which measure the correlation between the values of the series at different lags where the hypotheses are tested, were used for a value $\alpha = 0.05$, H_0 (the residuals are white noise), and H_a (the residuals are not white noise) (Ljung and Box, 1978). To verify normality in the residuals, the Jarque-Bera test was used, which compares the skewness and kurtosis of the series to those of a theoretical normal distribution with the hypotheses H_0 (there is normality in the residuals) and H_a (there is no normality in the residuals), for a value $\alpha = 0.05$ (Jarque and Bera, 1981).
The p values showed that there is insufficient evidence to reject the null hypothesis in the case of the Ljung-Box test, as the p values for all four models are greater than 0.05. Conversely, the Jarque-Bera test rejects the null hypothesis of normality in all models. None of the models that did not meet the assumption of normality in the residuals was discarded, and therefore, they were still considered for robustness.

Best ARIMA model by the Akaike index

The selection criterion is to choose the one with the lowest Akaike index score. In this case, the ARIMA (1,1,3)(1,0,1)[12] model was the best, with a value of -329.3, followed by ARIMA (3,1,1)(1,0,1)[12] with -328.43, ARIMA (1,1,0)(1,0,1)[12] with -326.36, and ARIMA (0,1,1)(1,0,1)[12] with -324.74.
The ARIMA (1,1,3)(1,0,1) model[12] can be written as follows:

$$(1-B)Y_t = \frac{(1+0.7911B-0.2539B^2+0.2266B^3)(1+0.9310B)}{(1-0.5257B)(1-0.9917B)}a_t,$$

where $\{a_t\}$ is white noise with zero mean and $\sigma^2 = 0.0094$, $\theta_1 = 0.7911$, $\theta_2 = 0.2539$, $\theta_3 = 0.2266$, $\theta_1 = -0.9310$, $\phi_1 = 0.5257$, and $\Phi_1 = 0.5257$, and $\Phi_1 = 0.9917$.

Best ARIMA model by the forecast error

For the selection of the best ARIMA model according to the estimation of the errors (RMSE, MAPE, and MASE) through cross-validation, the period from January 2006 to December 2018 was considered the training set. The coefficients of the models already selected up to this point were re-estimated, and the following three years were forecast. The forecasts were then compared with the original series to measure their accuracy. According to the methodology, the best model is the ARIMA (1,1,0)(1,0,1)[12], which has lower values in all adjustment metrics.

$$(1-B)\,ln(Y_t) = \frac{(1+0.9126B)}{(1+0.2505B)(1-0.9870B)}\,a_t,$$

where $\{a_t\}$ is white noise with zero mean and $\sigma^2 = 0.0098$, $\theta_1 = -0.9126$, $\phi_1 = -0.2505$, and $\Phi_1 = 0.9870$.

Forecasting in ARIMA models

The best models, according to the Akaike index and forecast error adjustment metrics, were ARIMA (1,1,3)(1,0,1)[12] and ARIMA (1,1,0)(1,0,1)[12], respectively. The forecast for both models were obtained using the R software (Figure 2).

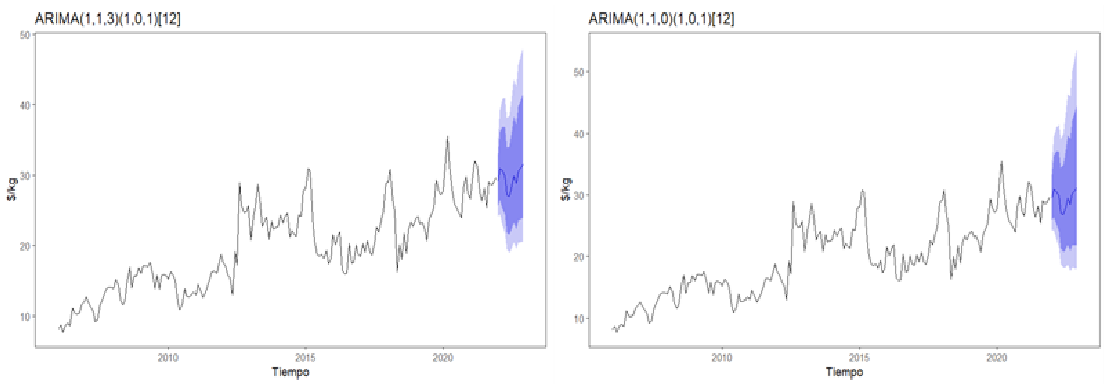

Figure 2. ARIMA model forecasting.

Regression model adjustment with ARIMA errors

Selection of candidate regression models with ARIMA errors

The same ARIMA models (Table 1) were proposed for regression models with ARIMA errors as found in the ACF and PACF plots (Figure 1). These models were used to estimate the error of a regression that included an exogenous variable, in this case the square root of egg imports into Mexico. These models were used to estimate the error of a regression that included an exogenous variable, in this case the square root of egg imports into Mexico.

Estimating coefficients of regression models with candidate ARIMA errors

In this case, the same logic was followed as in the ARIMA models, eliminating the models with parameters not significant at the level of 0.05 Only two models had all significant coefficients (regression with ARIMA (1,1,0)(1,0,1) errors[12] and regression with ARIMA (0,1,1)(1,0,1) errors[12]).

Diagnosis of regression models with candidate ARIMA errors

The same procedure as in the ARIMA models was used to diagnose the residuals and ensure that the model assumptions were met. Both models met the white noise criterion but failed to meet the residual normality criterion. As with the ARIMA models, the pair of models were kept to assess their accuracy.

Best regression model with ARIMA errors by Akaike Index

The best regression model with ARIMA errors, according to the Akaike criterion, is the regression model with ARIMA error (1,1,0)(1,0,1)[12], which presented the lowest value (-335.5), surpassing the ARIMA (1,1,0)(1,0,1)[12], which presented a value of -333.84. The regression model with ARIMA (1,1,0)(1,0,1) errors[12] can be written as follows:

$$(1-B)\ln(Y_t) = 0.0023\sqrt{X_t} + N_t$$

$$N_t = \frac{(1+0.8961B)}{(1+0.3215B)(1-0.9840B)}a_t,$$

where $\{a_t\}$ is white noise with zero mean and σ_2 = 0.0093, θ_1 = -0.8961, ϕ_1 = -0.3215, Φ_1 = 0.9840, and X_t = eggs imports in Mexico.

Best regression model with ARIMA errors by the forecast error

The best model according to the forecast error criterion is the regression model with ARIMA error (1,1,0)(1,0,1)[12], which presented the lowest value in all adjustment metrics, coinciding with the best model found by the Akaike index.

Forecasting in regression models with ARIMA errors

In this step, an ARIMA model with interventions to the exogenous variable was used to forecast the dependent variable based on the regression model. The ARIMA (0,1,3) error model was associated with the dynamic intervention model that best fitted the egg import time series data. The ARIMA (1,1,0)(1,0,1)[12] is the best error model associated with the regression based on the Akaike index and forecast error metrics (Figure 3)

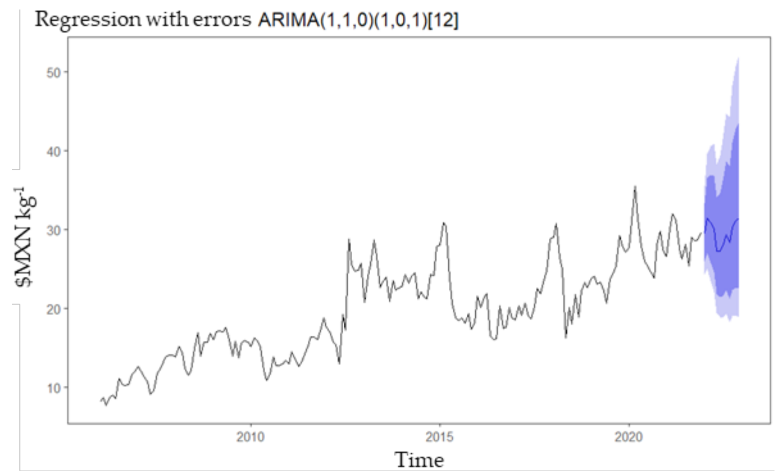

Figure 3. Forecasting the regression model with ARIMA errors.

ARIMA model adjustment with interventions

Identification of interventions

To identify changes in the series' level, the tso() function from the tsoutliers package in R was applied to the log-transformed series. Three interventions that changed the level of the series were detected in July and August 2012, as well as in May 2018. These interventions were modeled using dummy variables distributed in vectors that had values of zero before and one after the intervention.

The seasonality effect of the months of the year with the greatest variation in egg prices was also considered in the analysis. February, March, April, and June were discovered to have the greatest effect, and they were modelled with dummy variables that indicated one when the month occurred and zero when it did not.

Selection of ARIMA models with candidate interventions

The same ARIMA models identified (Table 1) were proposed according to the ACF and PACF plots (Figure 1), incorporating the effects of the interventions. In some models, the June effect was omitted because it was not significant.

Coefficient estimation for ARIMA models with candidate interventions

The same logic was followed as in the previous models. Only four models had all significant coefficients (ARIMA (3,1,2)(1,0,1)[12], ARIMA (2,1,3)(1,0,1)[12], ARIMA (1,1,0)(1,0,1)[12] and ARIMA (0,1,1)(1,0,1)[12] with interventions).

ARIMA model diagnosis with candidate interventions

The same procedure as in previous models was used to diagnose the residuals and ensure that they met the model's assumptions. In this case, all models met the criteria for residual normality and white noise.

Best ARIMA model with interventions by the Akaike index

According to the Akaike criterion, the best model was ARIMA (0,1,1)(1,0,1)[12] with interventions, which showed a value of -371.93, followed by ARIMA (1,1,0)(1,0,1)[12] with -371.81, ARIMA (3,1,2)(1,0,1)[12] with -371.23, and ARIMA (1,1,0)(1,0,1)[12] with -370.93. The ARIMA (0,1,1)(1,0,1)[12] model with interventions can be writte as follows:

$$(1 - B)\ln(Y_t) = 0.3793\,\xi_{1t} + 0.4324\,\xi_{2t} - 0.3111\,\xi_{3t} + 0.0826\,\xi_{4t} + 0.0790\,\xi_{5t} + 0.0712\,\xi_{6t} + N_t$$

$$N_t = \frac{(1 + 0.1701B)(1 + 0.9353B)}{(1 - 0.9866B)} a_t,$$

$$\xi_{1t} = \begin{cases} 0 & if\ T < 78 \\ 1 & if\ T \geq 78 \end{cases}$$

$$\xi_{2t} = \begin{cases} 0 & if\ T < 80 \\ 1 & if\ T \geq 80 \end{cases}$$

$$\xi_{3t} = \begin{cases} 0 & if\ T < 149 \\ 1 & if\ T \geq 149 \end{cases}$$

$$\xi_{4t} = \begin{cases} 0 & if\ T \neq feb \\ 1 & if\ T = feb \end{cases}$$

$$\xi_{5t} = \begin{cases} 0 & if\ T \neq mar \\ 1 & if\ T = mar \end{cases}$$

$$\xi_{6t} = \begin{cases} 0 & if\ T \neq abr \\ 1 & if\ T = abr \end{cases}$$

where, according to the Jarque-Bera test carried out on the residuals, $\{a_t\}\sim N\ (0,0.0076)$, $\theta_1 = -0.1701$, $\theta_1 = -0.9353$, and $\Phi_1 = 0.9866$.

Best ARIMA model with interventions by the forecast error

According to the forecast error criterion, the best model was the ARIMA (3,1,2)(1,0,1)[12] with interventions, which presented the lowest value in all metrics. The model can be written as follows:

$$\begin{aligned}(1 - B)\ln(Y_t) = {} & 0.4194\xi_{1t} + 0.4241\xi_{2t} - 0.2950\xi_{3t} \\ & + 0.0815\xi_{4t} + 0.0814\xi_{5t} + 0.0711\xi_{6t} \\ & - 0.0564\xi_{7t} + N_t\end{aligned}$$

$$N_t = \frac{(1 + 0.5628B - 0.9999B^2)(1 + 0.8717B)}{(1 - 0.3762B + 0.8527B^2 + 0.2158B^3)(1 - 0.9437B)} a_t,$$

$$\xi_{1t} = \begin{cases} 0 & if\ T < 78 \\ 1 & if\ T \geq 78 \end{cases}$$

$$\xi_{2t} = \begin{cases} 0 & if\ T < 80 \\ 1 & if\ T \geq 80 \end{cases}$$

$$\xi_{3t} = \begin{cases} 0 & if\ T < 149 \\ 1 & if\ T \geq 149 \end{cases}$$

$$\xi_{4t} = \begin{cases} 0 & if\ T \neq feb \\ 1 & if\ T = feb \end{cases}$$

$$\xi_{5t} = \begin{cases} 0 & if\ T \neq mar \\ 1 & if\ T = mar \end{cases}$$

$$\xi_{6t} = \begin{cases} 0 & if\ T \neq abr \\ 1 & if\ T = abr \end{cases}$$

$$\xi_{7t} = \begin{cases} 0 & if\ T \neq jun \\ 1 & if\ T = jun \end{cases}$$

where, according to the Jarque-Bera test, $\{a_t\}$~N (0,0.0075), θ_1 = -0.5628, θ_2 = -0.9999, θ_1 = -0.8717, ϕ_1 = 0.3762, ϕ_2 = -0.8527, ϕ_3 = -0.2158, and Φ_1 = 0.9437.

Forecasting ARIMA models with interventions

According to the Akaike index and forecast error metrics, ARIMA (0,1,1)(1,0,1)[12] and ARIMA (3,1,2)(1,0,1)[12] were found to be the best models associated with the error of the ARIMA (0,1,1)(1,0,1)[12] interventions. Model forecasts were obtained in R software (Figure 4).

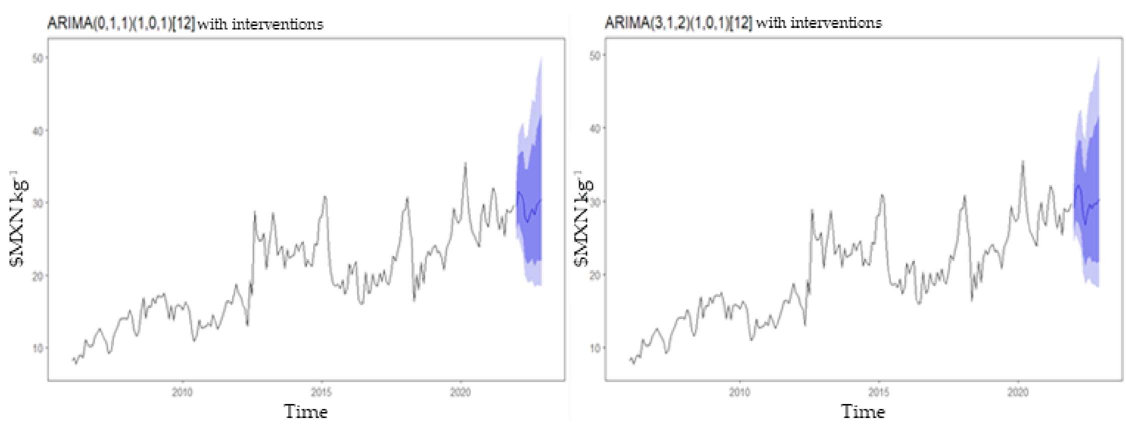

Figure 4. ARIMA model forecasting with interventions.

ARIMA model adjustment with transfer

Correlation graph of the dependent variable and predictor variable

The cross-correlation between the dependent variable (wholesale white egg price) and the predictor (egg imports) indicated that both variables are contemporaneous since there is a significant peak at lag zero. This indicates that there is no effect of lags of the predictor variable on the dependent variable, implying that the relationship between the variables does not change over time. Therefore, the analysis only considers the effect of the predictor variable without lags, which is precisely the definition of regression models with ARIMA errors. The adjustment results of the ARIMA models with transfer are the same as those of the linear regression model with ARIMA errors.

Comparison of methodologies according to Akaike index and forecast error metrics. According to the Akaike index (Table 2), the best model was the ARIMA (0,1,1)(1,0,1)[12] with interventions, followed by the regression models with ARIMA and ARIMA with transfer errors, which had the same adjustment, because they turned out to be the same model since the wholesale white egg price series and imports are contemporaneous.

Table 2. Comparison of methodologies by Akaike index.

	ARIMA	Regression with ARIMA errors	ARIMA with interventions
Akaike index	-329.3	-335.5	-371.9
Ljung-Box test *p*-value	0.9974	0.7763	0.7781
Jarque-Bera test *p*-value	6.929×10^{-6}	2.051×10^{-7}	0.8992
Variance	0.0094	0.0093	0.0076

According to the forecast errors (Table 3), the most accurate models were the regression model with ARIMA (1,1,0)(1,0,1) error[12] and the ARIMA (1,1,0)(1,0,1) error[12] with transfer since they are equivalent models. In both comparisons, only the ARIMA models with interventions meet the assumptions of white noise and normality in the residuals, in addition to having lower variance.
Luis-Rojas *et al*. (2019) found as the best model for egg price prediction the ARIMA (0,1,1)(1,0,1)[12], with an Akaike index of -332.538 and a variance of 0.0104. According

Table 3. Comparison of methodologies according to forecast error.

	ARIMA	Regression with ARIMA errors	ARIMA with interventions
RMSE	3.946	3.698	4.573
MAPE	10.91	10.12	13.04
MASE	0.8936	0.8245	1.071
Ljung-Box test *p*-value	0.7543	0.7763	0.8624
Jarque-Bera test *p*-value	2.719×10^{-11}	2.051×10^{-7}	0.8358
Variance	0.0098	0.0093	0.0075

to these authors, the model's short-term predictions deviate by 12.38 % from the observed data. However, they do not report specific data on their forecast errors. This result largely coincides with that found in the present study, since among the best prediction models is the ARIMA (0,1,1)(1,0,1)[12] with a slightly higher Akaike index (-329.3), which can be explained by the size of the series.

Furthermore, Luis-Rojas *et al.* (2022) obtained a time series model to forecast nominal monthly white egg prices paid to the producer in Mexico using transfer function models (TFM), evaluating their relationship with average rural sorghum prices. This model yielded an Akaike index of -352.97 and a variance of 0.0098. As can be seen, the ARIMA (0,1,1)(1,0,1)[12] model with interventions has a lower Akaike index (-371.9) and variance (0.0075).

CONCLUSIONS

The regression model with ARIMA errors turned out to be the same as the ARIMA with transfer, since the latter is a modification of the former and the lags of the predictor variable (egg imports into Mexico) had no significant effect on the response variable (wholesale white egg price in Mexico). According to Akaike's index, the model that presented the best adjustment was the ARIMA (0,1,1)(1,0,1)[12] with interventions, followed by the regression models with ARIMA errors and ARIMA with transfer. In the evaluation of forecast error, the best models were the regression models with ARIMA (1,1,0)(1,0,1)[12] and ARIMA (1,1,0)(1,0,1)[12] errors with transfer, followed by ARIMA with interventions. These models made it possible to generate reliable predictions of the price of white eggs in Mexico, which can help generate strategies that benefit in the short, medium, and long term.

According to the Akaike index and forecast error metrics, the ARIMA models with interventions presented low variance and met the assumptions of white noise and normality in the residuals. Furthermore, it can be concluded that the use of the ARIMA model with interventions, ARIMA with transfers, and regression with ARIMA errors improves the forecasting of the ARIMA model for the wholesale price of white eggs.

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REFERENCES

Akaike H. 1974. A new look at the statistical model identification. IEEE Transactions on Automatic Control 19 (6): 716–723. https://doi.org/10.1109/tac.1974.1100705

Alburquerque AP, de Moraes MC. 2007. Modelagem econométrica para una previsión del futuro del cacau: abordagem ARIMA. Revista Ciencias Administrativas 13 (2): 193–207.

Banco de México. 2022. Información histórica del mercado cambiario y de valores. Sistema de Información Económica. Banco de México. Ciudad de México, México. https://www.banxico.org.mx/SieInternet/consultarDirectorioInternetAction.do?accion=consultarCuadro&idCuadro=CE49&locale=es (Retrieved: June 2022).

Box GE, Jenkins G, Reinsel G, Ljung G. 2016. Time series analysis forecasting and control (Fifth edition). John Wiley and Sons: Hoboken, NJ, USA. 720 p.

Box GE, Tiao G. 1975. Intervention analysis with applications to economic and environmental problems. Journal of the American Statistical Association 70 (349): 70–79. https://doi.org/10.2307/2285379

Box GE, Jenkins GM. 1973. Some comments on a paper by Chatfield and Prothero and on a review by Kendall. Journal of the Royal Statistical Society. Series A (General) 136 (3): 337–352. https://doi.org/10.2307/2344995

Box GE, Pierce DA. 1970. Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. Journal of the American Statistical Association 65 (332): 1509–1526. https://doi.org/10.2307/2284333

Brockwell PJ, Davis RA. 2002. Introduction to time series and forecasting (Second edition). Springer: Cham, Switzerland. 425 p. https://doi.org/10.1007/978-3-319-29854-2

Cheung YW, Lai KS. 1995. Lag order and critical values of the augmented Dickey–Fuller test. Journal of Business and Economic Statistics 13 (3): 277–280. https://doi.org/10.2307/1392187

Diebold FX, Mariano RS. 2002. Comparing predictive accuracy. Journal of Business and Economic Statistics 20 (1): 134–144

Hanke JE, Wichern DW. 2010. Pronósticos en los negocios (Novena edición). Pearson: Naucalpan de Juárez, México. 576 p.

Hyndman RJ, Koehler AB. 2006. Another look at measures of forecast accuracy. International Journal of Forecasting 22 (4): 679–688. https://doi.org/10.1016/j.ijforecast.2006.03.001

Jarque CM, Bera AK. 1981. Efficient tests for normality, homoscedasticity, and serial independence of regression residuals: Monte Carlo evidence. Economics Letters 7 (4): 313–318. https://doi.org/10.1016/0165-1765(81)90035-5

Jiang F, Yang X, Li S. 2018. Comparison of forecasting India's energy demand using an MGM, ARIMA model, MGM-ARIMA model, and BP neural network model. Sustainability 10 (7): 2225. https://doi.org/10.3390/su10072225

Kim S, Kim H. 2016. A new metric of absolute percentage error for intermittent demand forecasts. International Journal of Forecasting 32 (3): 669–679. https://doi.org/10.1016/j.ijforecast.2015.12.003

Ljung GM, Box GE. 1978. On a measure of a lack of fit in time series models. Biometrika 65 (2): 297–303. https://doi.org/10.1093/biomet/65.2.297

Luis-Rojas S, García-Sánchez RC, García-Mata R, Arana-Coronado OA, González-Estrada A. 2019. Metodología Box-Jenkins para pronosticar los precios de huevo blanco pagados al productor en México. Agrociencia 53 (7): 665–678.

Luis-Rojas S, García-Sánchez RC, García-Mata R, Arana-Coronado OA, González-Estrada A. 2022. Modelo de función de transferencia para pronosticar el precio del huevo blanco, 2000-2017. Agricultura, Sociedad y Desarrollo 19 (2): 141–153. https://doi.org/10.22231/asyd.v19i2.1263

Marroquín-Martínez G, Chalita-Tovar L. 2011. Aplicación de la metodología Box-Jenkins para pronósticos de precios en jitomate. Revista Mexicana de Ciencias Agrícolas 2 (4): 573–577.

Reinsel GC. 1997. Elements of multivariate time series analysis (Second edition). Springer: New York, NY, USA. 375 p.

Reyes RT. 2014. Modelos ARIMA de las precipitaciones mensuales en el callejón de Huaylas (Perú). Aporte Santiaguino 7 (2): 47–56. https://doi.org/10.32911/as.2014.v7.n2.474

Sánchez-López E, Barreras-Serrano A, Pérez-Linares C, Figueroa-Saavedra F, Olivas-Valdez JA. 2013. Aplicación de un modelo ARIMA para pronosticar la producción de leche de bovino en Baja California, México. Tropical and Subtropical Agroecosystems 16 (3): 315–324.

SNIIM (Sistema Nacional de Información de Mercados). 2022. Precios de aves de corral. Gobierno de México. Secretaría de Economía. Ciudad de México, México. http://www.economia-sniim.gob.mx/nuevo/Home.aspx (Retrieved: June 2022).

Seijas C. 2002. Modelo estocástico de la serie de tiempo económica "inflación en Venezuela (Junio/95 a Junio/2000)". Revista INGENIERIA UC 9 (1): 1–12.

SIAP (Servicio de Información Agroalimentaria y Pesquera). 2022. Panorama Agroalimentario 2022. Gobierno de México. Secretaría de Agricultura y Desarrollo Rural. Servicio de Información Agroalimentaria y Pesquera. Ciudad de México, México. 217 p.

Tsay RS. 2005. Analysis of financial time series (Second edition). Jhon Wiley and Sons: Hoboken, NJ, USA. 638 p.

Velicer WF. 1976. Determining the number of components from the matrix of partial correlations. Psychometrika 41 (3): 321–327. https://doi.org/10.1007/BF02293557